**Introduction**

Graphic designers use mathematical models to plan the animation of water fountains. The task given is to find a function which models the shape of a water spout and produce a report explaining how this function was found. In this report various methods will be used to create polynomial functions which appear to follow the arc of the water spout.

**Assumptions**

* The photograph taken of the water stream is taken perpendicular to the camera because care was made to find a right angle between the lens and the water stream.
* Factors effecting the shape of the stream including wind, water pressure and vibration were controlled. This was assumed because it was taken undercover, the valves in drinking fountain are designed to provide constant pressure and it was taken in still conditions.

**Observations**

* The shape of the water stream resembles a parabolic function because the water has an upwards initial velocity generated by the water pressure and is then affected by gravity which makes it move downwards and is an example of parabolic projectile motion.
* The shape of the water stream was wavier and less consistent the further the water went from the origin because the water had more time to be affected by the air around it.

**Mathematical Translation**

While solving the task, various mathematical concepts will be used. A quadratic is a mathematical expression with a power of two. It is usually written as a Cartesian equation using a general form4

. For the purpose of this report a rearranged version the general quadratic form called the turning point form which is written as . The turning point of a quadratic is defined as its vertex or where the graph “turns around” (Characteristics of Parabolas, 2023). In the quadratics turning point form this turning point is can be expressed as the coordinates . The last variable in the formula is . This impacts the dilation and reflection of the graph. \*find in textbook\*. As this number moves further from zero the graph becomes narrower. Whether the graph is reflected is determined by whether the graph is negative or not. A negative graph has a maximum turning point and approaches negative infinity while a positive graph has a minimum turning point and approaches positive infinity. Another separate mathematical concept of an r2 value also known as the coefficient of determination. It is a measure that assesses the ability of a model to predict or explain an outcome (Enders, 2022). In the context of this report it describes how closely a function correlates with a set of points.

* Polynomial – A maths equation with powers higher than one
* Quadratic – A polynomial with a highest power of two
* Turning point – The point which a quadratic flips its direction and is either a maximum or minimum
* Dilation – The coefficient of a quadratics x2 term. As this number moves further from zero the graph becomes narrower
* Reflection – The property of a quadratic describing whether or not it extends positively or negatively from the turning point. Is determined by whether or not the x2 coefficient is positive or negative
* r2 value – A value which describes the correlation of a function to defined data point. As it approaches its limit of one the correlation increases.
* Quadratic general form – the standard and most expanded way of writing a quadratic
* Quadratic turning point form – A way of rearranging the quadratic formula so that is the location of the turning point

**Mathematical Calculations**

To begin the investigation a photo was taken of the water spout using a digital camera (figure 1).



Figure 1: Photograph of water spout

This was achieved by ensuring that the camera was square to the stream, the photo includes the water from the beginning of the stream to the just below the sink edge.

The photo was then printed and by using graph paper the following points were added to the Cartesian plane (figure 2)

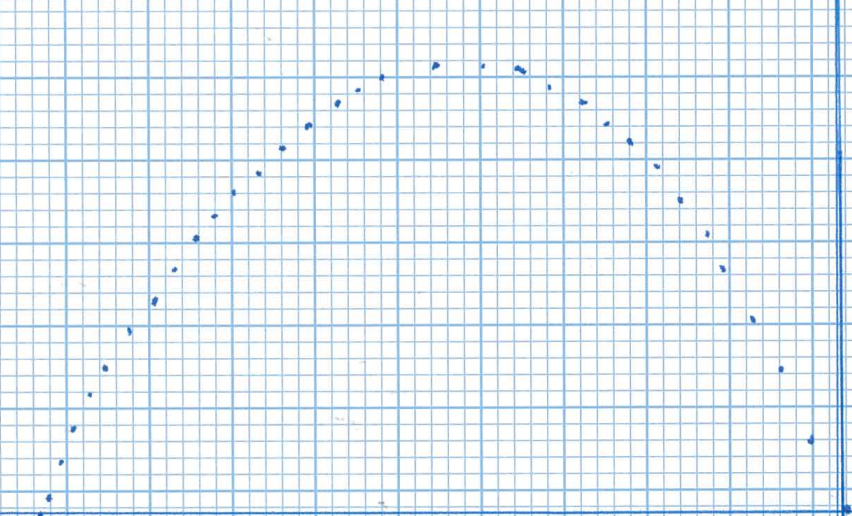


Figure 2 Points of the top side of the water spout

This was achieved by overlaying graph paper with the printed photograph and plotting the points using the start of the fountain as the origin.

By inspecting the dots on the page, a list of coordinates was then created (figure 3)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | y |  | 27.5 | 25 |
| 0 | 0 |  | 28.5 | 24.5 |
| 2 | 4 |  | 30.5 | 23 |
| 4 | 8 |  | 32 | 22 |
| 5.5 | 11 |  | 33.5 | 20.5 |
| 7.5 | 14.5 |  | 35 | 19 |
| 8 | 16.5 |  | 36 | 17.5 |
| 10 | 18.5 |  | 37 | 16 |
| 11.5 | 20.5 |  | 38.5 | 14.5 |
| 13 | 22 |  | 39.5 | 12.5 |
| 14.5 | 23 |  | 41 | 11 |
| 16 | 24.5 |  | 42.5 | 8.5 |
| 18 | 25.5 |  | 43.5 | 7 |
| 20 | 26.5 |  | 44.5 | 5 |
| 21.5 | 27 |  | 45 | 3 |
| 23 | 26.5 |  | 45.5 | 1 |
| 26 | 26 |  | 46 | 0 |

Figure 3: List of coordinates derived from graph paper

1. Using the turning point form of a quadratic and algebra the following function was created
2. Using Excel and the table of points a chart was created and a quadratic and quartic function with r2 values

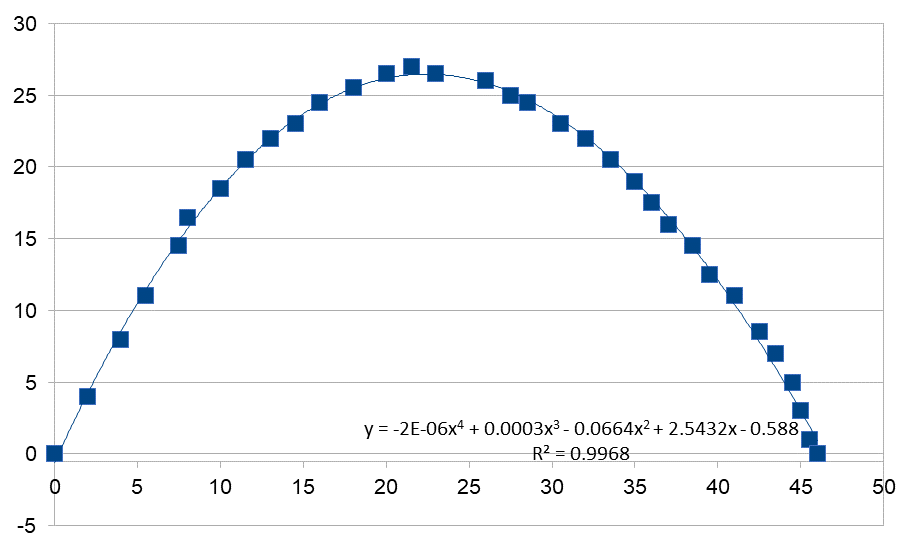


Figure 4: Graph of points using quartic trendline in Excel

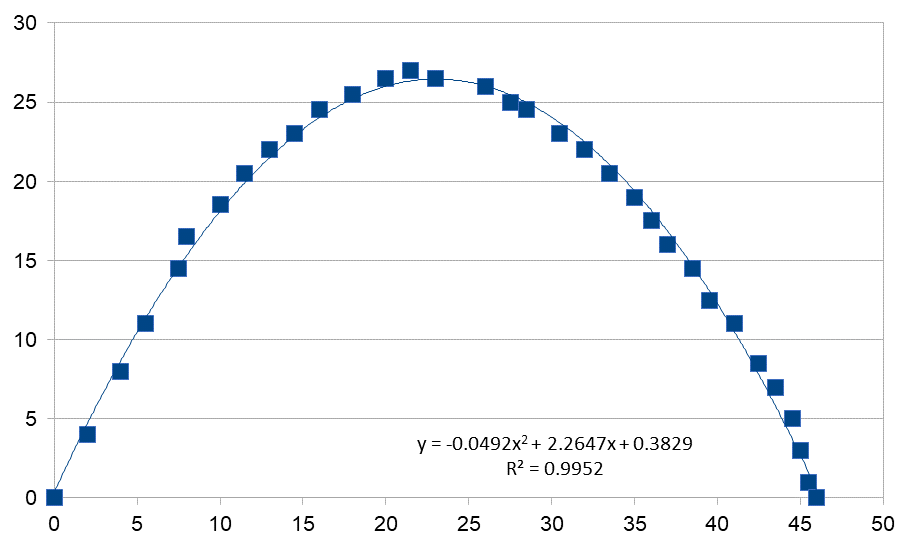


Figure 5: Graph of points using quadratic trendline in Excel

1. Using demos, I inputted the list of coordinates and functions created by steps 5 and 4

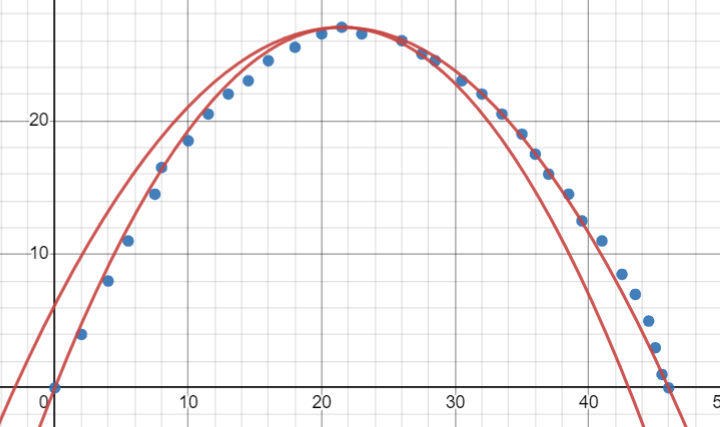


Figure 6: Graph of points and quadratics derived from step 4

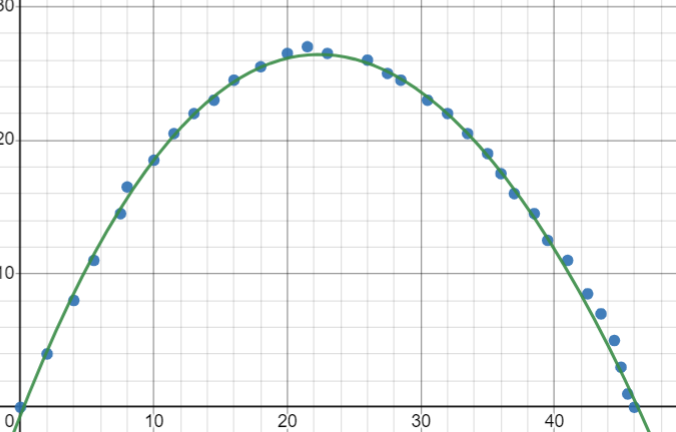


Figure 7: Graph of points and quartic derived from step 5

1. After observing the output of Desmos restrictions were added to the domain of the functions using

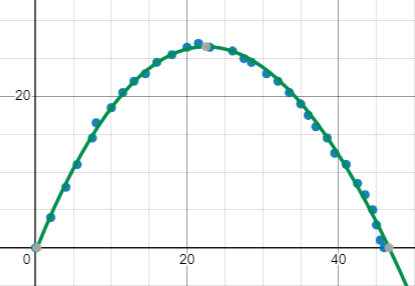


Figure 8 Graph of points and quartic with restricted domain

This was done because the values of x which less than one are not valid as they are before the stream started and should not be graphed.

The solution which matches closest to the drinking fountain was the quartic created by the trendline feature of Microsoft Excel.

This is the most valid solution because it was the largest correlation value and therefore matches the closest to the true line created by the water spout

**Evaluate**

|  |  |
| --- | --- |
| **Assumption** | **Evaluation** |
| The photograph taken of the water stream is taken perpendicular to the camera because care was made to find a right angle between the lens and the water stream. | Impacted the solution because without assuming this the function would not be accurate to the true water spout |
| There are controlled factors effecting the shape of the stream, these include wind, water pressure and vibration. This was assumed because it was taken undercover, the valves in water bubblers are designed to provide constant pressure and it was taken in still conditions. | Impacted the solution because it makes the functions closer to a true parabola.  This is important because the closer to a parabola the shape of the water is means the polynomial generated can be closer to the true shape. |

|  |  |
| --- | --- |
| **Observation** | **Evaluation** |
| The shape of the water stream resembles a parabolic function because the water has an upwards initial velocity generated by the water pressure which causes it to move upwards and is then affected by gravity which makes it move downwards | Impacted the solution because it made polynomials with power of 2-4 the most effective way to create a function of the water spout  This is important because it is the topic of the report |
| The shape of the water stream was wavier and less consistent the further the water went from the origin because the water | Impacted the solution because it made it more difficult to accurately find the outside of the edge of the water stream towards the end of the stream |

Strengths

* The solution used polynomials to find a function rather than a more complicated function such as trigonometric or piecewise function
* The function generated has a very high correlation with the original dot as show by a high r2 value

Limitations:

* Both the accuracy of the points manually drawn and reading those points had some amount of error causing the functions to not entirely match the true water stream
* Due to using a photo taken from one angle there was some perspective error caused by the image not reflecting the true 2d cross section of the stream

**Conclusion**

The formula both visually matches the points drawn on the graph paper and has the highest r2 value so therefore is the function most accurate to the picture of a water spout.